

Impulse Differential Equations of Environmental Problems

Yuldashova Hilola Ataxanovna, Rashidov Sardor Gulomovich

Lecturer at Nukus State Pedagogical Institute

Abstract:

When the number of microorganisms reaches a certain amount, \tilde{x} say, when they reach a certain number, the number of deaths will increase sharply in a short period of time. If the number of deaths is a fraction of the number $\frac{1}{n}$, ($n > 1$) of living organisms, then the law of change in the number of organisms in a colony

$$\frac{dx}{dt} = kx - \lambda x^2, \quad t \notin D,$$

$$\Delta x|_{t \in D} = \frac{1}{n} x(t) - x(t) = \frac{1-n}{n} x(t),$$

$$x(t_0) = x_0$$

studied to be determined by the solution of the Cauchy problem for a differential equation with a momentum effect in the form. What we are D talking about here $x(t)$ is a μ set of moments in time when \tilde{x} the number of microorganisms is t less than.

The number of microorganisms under consideration is called the Malthus model

$$x(t) = x_0 e^{k(t-t_0)}$$

legitimacy is developed.

Simply put, for a species living in a real natural environment for reasons such as microbial habitat and food deprivation as well as the spread of infection, their modification problems were studied using the Cauchy problem posed for differential equations with impulse effects.

Keywords: Differential equation with impulse effect, Cauchy problem, Malthus model, growth rate, Ferxulsta-Perla model, logistic curve.

Any species, say a species of microorganism, lives in a specific (or artificial species) environment, where there are no species other than the species. Due to the laws of nature, the spontaneous growth and decline of this species of living microorganisms change over time, and the rate of change is proportional to the number of adults in them.

Let the number $x(t)$ of living organisms be t in time $t + \Delta t$ and $x(t + \Delta t)$ in time. It shows the Δt change in the number of these microorganisms under consideration over time

$$\Delta x = x(t + \Delta t) - x(t)$$

will be During this time, adult microorganisms, or parts of them, increase in number, and some die. To change the number of microorganisms in it

$$\Delta x = N - M \quad (1)$$

where we mean N the number of Δt microorganisms born in a time interval, and M we mean the Δt number of microorganisms killed in a time interval.

The number of N microorganisms that are Δt born depends on the time interval and the Δt number of parents, i.e., the larger the time interval, the greater the number of microorganisms of similar age. the number of microorganisms and so on. Therefore for N svaasi

$$N = F(x, \Delta t)$$

it is possible F to accept the x expression, where the function of which Δt increases with the increase of or, is a function which becomes zero when one of these two variables is equal to zero. According to this F definition of the function

$$N = k_1 x \Delta t \quad (2)$$

can be expressed in the form, where k_1 the proportionality coefficient.

For the number of microorganisms that kill a M similar species

$$M = k_2 x \Delta t \quad (3)$$

formula.

To put (2) and (3) in their places in equation (1)

$$\Delta x = k_1 x \Delta t - k_2 x \Delta t$$

or

$$\Delta x = (k_1 - k_2) x \Delta t = k x \Delta t \quad (4)$$

where the growth rate of the $k = k_1 - k_2$ microorganism itself is obtained.

In the real world, lumbering elephants are exposed x , M by the aggression N of speeding midgits. Therefore, it is difficult to say anything about the continuity of a function and its differentiation. However, the colony of microorganisms is large, and if the time interval is Δt small, then the function is reminiscent $x(t)$ of a continuous function in its nature.

(4) Divide both sides Δt of the equation by e and $\Delta t \rightarrow 0$ take the limit

$$\lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} = kx$$

or

$$\frac{dx}{dt} = kx$$

we have a differential equation in the form

We know that there is a general solution to this $x = Ce^{kt}$ equation. Agar the initial time t_0 of observation of the colony and the living at that time the number of microorganisms is called $x_0 = x(t_0)$ the Malthus model of $C = x_0 e^{-kt_0}$ the number of microorganisms under consideration

$$x(t) = x_0 e^{k(t-t_0)}$$

we will have legitimacy.

Now let's look at the law of change for a species that lives in real natural conditions due to the spread of infection, along with the lack of microorganisms, habitat, and food.

Due to self-poisoning, the growth Δx of the species is reduced to the level that $h(x, \Delta t)$ characterizes the true living conditions of the colony. Hence the equation (4) above

$$\Delta x = kx\Delta t - h(x, \Delta t) \quad (5)$$

will be visible. Without any other $h(x, \Delta t)$ error it can Δt be assumed that the svaasin is linearly connected, i.e.

$$h(x, \Delta t) = f(x)\Delta t$$

where is the $f(x)$ quadratic function, i.e., the $f(x) = \lambda x^2$ coefficient of intra-species intoxication (or intra-species intrusion). Unda

$$h(x, \Delta t) = \lambda x^2 \Delta t \quad (6)$$

(6) is significant $kx\Delta t$ for large s compared to svaa, and x for small x s it is infinitely small svaa, however the poisoning is not spontaneous, but due to the meeting of two organisms in the colony, and the number of encounters increases. The number of reciprocal struggles increases with time, that is, (6) $x \cdot x$ is proportional x^2 to svaa, hence qa. Thus, in this case, equation (6)

$$\Delta x = kx\Delta t - \lambda x^2 \Delta t \quad (7)$$

will be visible.

(7) Divide both sides of the Δt equation by and $\Delta t \rightarrow 0$ take the limit

$$\frac{dx}{dt} = kx - \lambda x^2$$

we have a differential equation in the form

Let's change the last equation and $\frac{k}{\lambda} = \mu$ enter the notation

$$\frac{dx}{dt} = kx \left(1 - \frac{\lambda}{k} x \right) = kx \left(\frac{\frac{k}{\lambda} - x}{\frac{k}{\lambda}} \right) = kx \left(\frac{\mu - x}{\mu} \right)$$

will be By separating the variables of the resulting equation

$$\left(\frac{1}{x} + \frac{1}{\mu - x} \right) dx = k dt$$

In terms of appearance, it is

$$\frac{x}{\mu - x} = C e^{kt}$$

has a general integral in the form

If we say the initial time of observation of the t_0 colony and the number of living microorganisms $x_0 = x(t_0) < \mu$ at that time, then the number $C = \frac{x_0}{\mu - x_0}$ of microorganisms under consideration

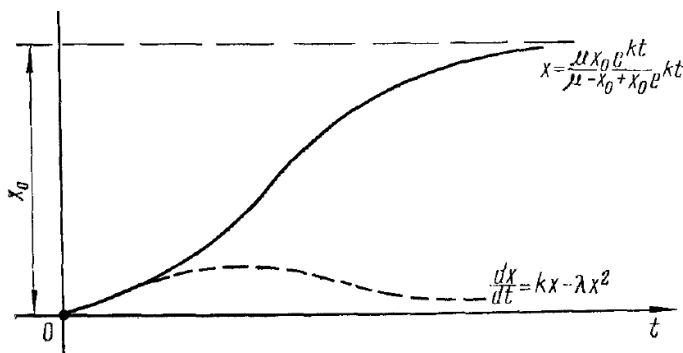
$$\frac{x}{\mu - x} = \frac{x_0}{\mu - x_0} e^{kt}$$

or so-called Ferxyulsta-Perla law

$$x(t) = \frac{\mu x_0 e^{kt}}{(\mu - x_0) e^{kt_0} + x_0 e^{kt}} \quad (8)$$

we have a solution.

The graph of this solution is called the logistics curve.



Now, we come to the part where we talk about the middle ground. When the number of microorganisms reaches a certain level, say, when they reach a certain number, the number of people who die due to the lack of habitat and food, as well as the spread of infection, can suddenly increase in a short period of time. If the number of deaths is a fraction of the $\frac{1}{n}$, ($n > 1$) number of living organisms, then the law of change in the number of organisms in a colony

$$\frac{dx}{dt} = kx - \lambda x^2, \quad t \notin D, \quad (9)$$

$$\Delta x|_{t \in D} = \frac{1}{n} x(t) - x(t) = \frac{1-n}{n} x(t), \quad (10)$$

$$x(t_0) = x_0 \quad (11)$$

is determined by the solution of the Cauchy problem for a differential equation with an impulse effect in the form D of If we assume that the $x(t)$ Cauchy problem μ is meaningless \tilde{x} for the differential t equation with impulse $\tilde{x} \geq \mu$ effect (9) - (11), in other words, the problem is not affected by the impulse.

We know that in the $t = t_0$ time interval from time to the effect of the first pulse, the solution of the Cauchy problem (9) - (11) is given by the formula (8), i.e.

$$x(t) = \frac{\mu x_0 e^{kt}}{(\mu - x_0) e^{kt_0} + x_0 e^{kt}}, \quad t_0 \leq t$$

determined by the formula

The problem is that when the number $x(t)$ of microorganisms \tilde{x} reaches a certain level, that is

$$\frac{\mu x_0 e^{kt}}{(\mu - x_0) e^{kt_0} + x_0 e^{kt}} = \tilde{x}$$

satisfying equality

$$t = \frac{1}{k} \ln \left| \frac{\tilde{x}(\mu - x_0)}{(\mu - 1)x_0} \right| e^{kt_0} = t_1$$

time (9) first receives the effect of the impulse, and the law of change of the number of microorganisms in the following times

$$\frac{dx}{dt} = kx - \lambda x^2, \quad t \notin D, \quad (12)$$

$$\Delta x|_{t \in D} = \frac{1}{n} x(t) - x(t) = \frac{1-n}{n} x(t), \quad (13)$$

$$x(t_1) = x_1 \quad (14)$$

is determined by the solution of the Cauchy problem for the differential equation with the impulse effect in the form, where

$$x_1 = \frac{(n-1)\mu x_0 e^{kt_1}}{n((\mu - x_0)e^{kt_0} + x_0 e^{kt_1})}$$

The law of change of the number of microorganisms after the first pulse, ie the solution of the Cauchy problem for the differential equation with the pulse effect (12) - (14) is determined by the formula and when the number $x(t)$ of microorganisms \tilde{x} reaches the amount again, ie

$$x(t) = \frac{\mu x_1 e^{kt}}{(\mu - x_1)e^{kt_1} + x_1 e^{kt}}, \quad t_1 \leq t$$

satisfying equality

$$\frac{\mu x_1 e^{kt}}{(\mu - x_1)e^{kt_1} + x_1 e^{kt}} = \tilde{x}$$

time (9) receives the second pulse or equation (12) the first pulse effect, and the law of change of the number of microorganisms $x(t)$ in the following periods

$$x(t) = \frac{\mu x_2 e^{kt}}{(\mu - x_2)e^{kt_2} + x_2 e^{kt}}, \quad t_2 \leq t$$

determined by the formula, here

$$x_1 = \frac{n\mu x_1 e^{kt_2}}{(n-1)((\mu - x_1)e^{kt_0} + x_1 e^{kt_2})}$$

and so on.

Continuing the process, Equation (9) is the solution after being subjected to a pulse several times, i.e.

$$t = \frac{1}{k} \ln \left| \frac{\tilde{x}(\mu - x_{m-1})}{(\mu - 1)x_{m-1}} \right| e^{kt_{m-1}} = t_m$$

the law of change of the number of microorganisms over time

$$x(t) = \frac{\mu x_m e^{kt}}{(\mu - x_m)e^{kt_m} + x_m e^{kt}}, \quad t_m \leq t$$

determined by the formula

References

1. Andronov A. A., Witt A. A., Haykin S. E. The theory of kuebaniy. –M.: Nauka, 1981. p.568.
2. Boguyubov N. N., Mitropuskiy YU. A. Asymptotic methods v teorii nelineynix kuebaniy. –M.: Fizmatgiz, 1963. p. 410.
3. Krasnoselskiy M.A. The sdviga operator on trajectories differential level. –M.: Nauka, 1966. p. 332.
4. Krasnoselskiy M.A., Vaynikko G.M., Zabreyko P.P. and dr. Priblijennoe reshenie operatornix uravdaniy. –M.: Nauka, 1969. p. 300.
5. Krilov N.M., Boguyubov N.N. Introduction to nonlinear mechanics. Kiev, AN USSR, 1937. p. 364. ▭
6. Lando YU.K. Kraevaya zadacha for lineynix integro-differentialnix uravdaniy Fredguma vtorogo roda. Izv. AN BSSR, ser. phys. Techn. Science. №4. 1960. p. 11-21.
7. Mitropuskiy YU.A. Method usreddaniya v nelineynoy mechanics.–Kiev, “Naukova dumka”, 1971. p. 440.
8. Mishkis A.D. The basic theorems of theories of obiknovennix differential uravdaniy v neklassicheskix sluchayax. First letnyaya matematicheskaya shkua. –Kiev, “Naukova dumka”, 1964.